Generation current temperature scaling

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The current per unit area generated inside the depleted bulk can be written as

\[ J = \frac{q W n_i}{\tau_g} \]  

(1)

where \( q \) is elementary charge, \( W \) – depleted thickness, \( n_i \) – intrinsic carrier concentration and \( \tau_g \) – generation lifetime. Temperature dependence of \( n_i \) can be expressed as

\[ n_i(T) \propto T^{3/2} \exp\left(-\frac{E_g}{2kT}\right) \]  

(2)

where \( T \) is the absolute temperature, \( k \) - the Boltzman constant and \( E_g \) - the band gap. Weak temperature dependence of the electron and hole effective masses is neglected in Eq.(2).

Assuming generation happening via a specific trap with density \( N_t \) and level \( E_t \) in the band gap the generation lifetime can be written as

\[ \tau_g = \frac{1}{N_t} \left( \frac{\exp\left(\frac{\Delta}{kT}\right)}{v_{tn} \sigma_n} + \frac{\exp\left(\frac{-\Delta}{kT}\right)}{v_{tp} \sigma_p} \right) \].  

(3)

Here \( \Delta = E_t - E_i \) is the difference between the trap level and the intrinsic Fermi level, \( v_{tn(p)} \) is the thermal velocity and \( \sigma_{tn(p)} \) the trapping cross-section for electrons (holes). The dependence of \( \tau_g \) on \( \Delta \) is close to \( \cosh(\Delta/kT) \) (assuming \( v_{tp} \sigma_p \approx v_{tn} \sigma_n \)). The minimum \( \tau_g \) is reached and thus the most effective current generation happens when the trap level is close to \( E_i \). In this case the temperature dependence of \( \tau_g \) is the simplest: \( \tau_g \propto T^{-1/2} \), and due only to that of the thermal velocities. (The temperature dependence of the effective carrier masses is again neglected.) For \( |\Delta| > 1.5 \) the additional factor in \( \tau_g \) temperature dependence is reduced to \( \exp(|\Delta|/kT) \). Therefore the temperature dependence of the generation current is usually parameterised as

\[ I(T) \propto T^2 \exp\left(-\frac{E_g + 2\Delta}{2kT}\right) \].  

(4)
where \( \Delta \) is a parameter close to the absolute value of \( \Delta_c \). An excess of the energy in the exponent over the \( E_g \) indicates the generation proceeding via a trap level noticeably different from \( E_i \).

The temperature dependence of the current is clearly dominated by that of \( n_i \). Thus it is crucial to know \( n_i(T) \) in detail. The intrinsic carrier concentration is a parameter of prime importance in semiconductor physics and a vast literature exists about it. This Note relies on a relatively recent Review [2] and concentrates on the temperature interval of \( \pm30^\circ C \) most relevant to the present usage of silicon detectors in Particle Physics.

Ref. [2] quotes three fits of \( n_i(T) \) found in different experimental studies. They are described by Eqs. (21, 22, 23) of that paper and have the form of

\[
n_i(T) \propto T^m \exp\left(-E_a/kT\right)
\]

where \( E_a \) is so called activation energy. In Eqs. (21, 22) \( m \) is set to the standard value of 3/2 and the results can be directly compared with parameterisation (2). In eq. (23) \( m \) is a free parameter of the fit and has the value of 2.365. At any temperature, \( T \), the dependence (5) can be converted to the equivalent one with \( m = 3/2 \) and activation energy \( E_{a_{eq}} \). Denote \( E_{a_i}^m \) the activation energy for the parameterisation (5) with specific \( m \) and require that the relative derivative of it is equal to that with \( m = 3/2 \). As a result one gets

\[
\frac{(dn_i/n_i)}{(dT/T)} = \frac{3}{2} + \frac{E_{a_{eq}}}{kT} = m + \frac{E_{a_i}^m}{kT}
\]

from which it follows

\[
E_{a_{eq}} = E_{a_i}^m + (m-3/2) kT
\]

In Eq. (23) of Ref.[2] the activation energy is \( k*6733K = 0.580 \text{ eV} \) and \( m=2.365 \). Using the relation (6) one obtains for \( T=273K \) the equivalent activation energy of 0.601 eV. It is easy to check that in the interval from -30° to +30°C the temperature dependences (5) for \( m = 2.365 \), \( E_a = 0.580 \text{ eV} \) and for \( m=3/2 \), \( E_a = 0.601 \text{ eV} \) differ by less than 1%.

The activation energy values in Eqs. (21, 22) of Ref.[2] are 0.605 and 0.603 eV respectively. Combining them with the value of 0.601 eV obtained above one gets the average experimental value of \( E_a = 0.603 \pm 0.002 \text{ eV} \) where the uncertainty covers all
three experimental values. Therefore the experimental value for the effective gap energy is

\[ E_{ef} = 2E_a = 1.206 \pm 0.004 \text{ eV}. \]  

(7)

This result looks incompatible with the experimental values of \( E_g \), which according to Table 1 of Ref.[2] are 1.1242 eV at 300K and 1.1367 eV at 250K. Note however that temperature independent \( E_{ef} \) should also incorporate temperature dependence of the band gap \( E_g \).

Consider the exponential term of Eq.(2)

\[ f(T) \propto \exp(-E_g/2kT) \]  

(8)

with \( E_g \) itself a function of temperature. The relative gradient of this function is

\[ (df/f)/(dT/T) = E_g(T)/2kT - (dE_g/dT)/2k \]  

(9)

If the temperature dependent \( E_g \) is replaced in Eq.(8) by a constant parameter \( E_{ef} \) then the relative gradient will be

\[ (df/f)/(dT/T) = E_{ef}/2kT \]  

(10)

Requiring the gradients given by Eqs. (9) and (10) to be equal at a given temperature, \( T \), one gets for \( E_{ef} \):

\[ E_{ef} = E_g(T) - T (dE_g/dT) \]  

(11)

Consider now the situation when in some temperature interval the band gap dependence can be approximated by a linear function

\[ E_g(T) = E_0 - \alpha T. \]  

(12)

Here \( E_0 \) is the band gap value extrapolated to \( T = 0 \text{K} \). In this case the effective energy gap found from Eq. (11) is

\[ E_{ef} = E_0 - \alpha T - T (-\alpha) = E_0 \]  

(13)

independent of \( T \) (and also of \( \alpha \)). This conclusion can be verified by direct substitution of \( E_g \) in Eq.(8) by relation (12):

\[ A \cdot \exp(-E_g(T)/2kT) = A \cdot \exp(-E_0/2kT + \alpha/2k) = A' \cdot \exp(-E_0/2kT) \]  

(14)

For temperatures above 250K and below 415K the parameterisation (12) is readily available in Ref. [2], Eq.(17). The value of \( E_0 \) in this equation is 1.206 eV in perfect agreement with the experimentally found \( E_{ef} \) presented in eq. (7). Thus there is no contradiction between the temperature dependent band gap energy and the temperature independent effective energy. To some extent surprisingly the effective
gap value happens to be outside the range of the actual band gap values in the considered temperature interval.

For non-irradiated sensors it is usually assumed that the current is generated via traps with energy levels near the mid gap i.e. the \( I(T) \) is described by Eq.(4) with \( \Delta=0 \) and either temperature dependent \( E_g(T) \) or the effective gap \( E_0 \). However in non-irradiated sensors the bulk generation current is typically quite small and presents no practical problems. The high current in such sensors is usually due to other reasons e.g. soft breakdown, which makes the above analysis irrelevant. It is more appropriate for irradiated sensors where the bulk current often dominates. The information on \( I(T) \) for irradiated sensors is rather scarce. The survey [3] made in 1994 produced the effective gap value of \( 1.24 \pm 0.06 \) eV, which agrees with the mentioned above \( E_0 = 1.21 \) eV for \( n_i \), but has a substantial uncertainty. The study of ATLAS SCT sensors irradiated by \( \sim 3 \times 10^{14} \) 24 GeV protons/cm\(^2\) performed in 1997-99 resulted in the effective gap value of \( 1.21 \) eV [4]. Thus until proven otherwise the same temperature dependence with \( E_{\text{ef}} \) equal to \( 1.21 \) eV may be used for both irradiated and non-irradiated sensors.

In conclusion, the temperature dependence of the bulk generation current can be described as

\[
I(T) \propto T^2 \exp(-1.21eV/2kT)
\]

both for non-irradiated and irradiated sensors. The difference between the effective energy value and the actual gap energy is due to using temperature independent \( E_{\text{ef}} \) instead of the temperature dependent \( E_g(T) \).

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References
4. ATLAS SCT Barrel Module FDR/2001, SCT-BM-FDR-7, p.19: \( E_{\text{ef}}/2k = 7019 \text{K} \), from which it follows: \( E_{\text{ef}} = 1.21 \) eV.