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Generation current temperature scaling

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The current per unit area generated inside the depleted bulk can be written as

$$J = \frac{qWn_i}{\tau_g} \qquad (1)$$

where q is elementary charge, W – depleted thickness, n_i – intrinsic carrier concentration and τ_g – generation lifetime. Temperature dependence of n_i can be expressed as

$$n_i(T) \propto T^{3/2} exp(-E_g/2kT) \qquad (2)$$

where T is the absolute temperature, k - the Boltzman constant and E_g - the band gap. Weak temperature dependence of the electron and hole effective masses is neglected in Eq.(2).

Assuming generation happening via a specific trap with density N_t and level E_t in the band gap the generation lifetime can be written as [1]

$$\tau_g = \frac{1}{N_t}\left(\frac{\exp\left(\frac{\Delta_t}{kT}\right)}{v_{tp}\sigma_p} + \frac{\exp\left(-\frac{\Delta_t}{kT}\right)}{v_{tn}\sigma_n}\right). \qquad (3)$$

Here $\Delta_t = E_t - E_i$ is the difference between the trap level and the intrinsic Fermi level, $v_{tn(p)}$ is the thermal velocity and $\sigma_{n(p)}$ the trapping cross-section for electrons (holes). The dependence of τ_g on Δ_t is close to $\cosh(\Delta_t/kT)$ (assuming $v_{tp}\ \sigma_p \approx v_{tn}\ \sigma_n$). The minimum τ_g is reached and thus the most effective current generation happens when the trap level is close to E_i. In this case the temperature dependence of τ_g is the simplest: $\tau_g \propto T^{-1/2}$, and due only to that of the thermal velocities. (The temperature dependence of the effective carrier masses is again neglected.) For $|\Delta_t| > 1.5$ the additional factor in τ_g temperature dependence is reduced to $exp(|\Delta_t|/kT)$. Therefore the temperature dependence of the generation current is usually parameterised as

$$I(T) \propto T^2 \exp\left(-\frac{E_g + 2\Delta}{2kT}\right). \qquad (4)$$

where Δ is a parameter close to the absolute value of Δ_t. An excess of the energy in the exponent over the E_g indicates the generation proceeding via a trap level noticeably different from E_i.

The temperature dependence of the current is clearly dominated by that of n_i. Thus it is crucial to know $n_i(T)$ in detail. The intrinsic carrier concentration is a parameter of prime importance in semiconductor physics and a vast literature exists about it. This Note relies on a relatively recent Review [2] and concentrates on the temperature interval of ±30°C most relevant to the present usage of silicon detectors in Particle Physics.

Ref. [2] quotes three fits of $n_i(T)$ found in different experimental studies. They are described by Eqs. (21, 22, 23) of that paper and have the form of

$$n_i(T) \propto T^{\,m} \exp(-E_a/kT) \qquad (5)$$

where E_a is so called activation energy. In Eqs. (21, 22) m is set to the standard value of 3/2 and the results can be directly compared with parameterisation (2). In eq. (23) m is a free parameter of the fit and has the value of 2.365. At any temperature, T, the dependence (5) can be converted to the equivalent one with m=3/2 and activation energy E_a^{eq}. Denote E_a^m the activation energy for the parameterisation (5) with specific m and require that the relative derivative of it is equal to that with m =3/2. As a result one gets

$$(dn_i/n_i)/(dT/T) = 3/2 + E_a^{eq}/kT = m + E_a^{m}/kT$$

from which it follows

$$E_a^{eq} = E_a^{m} + (m-3/2)\, kT \qquad (6)$$

In Eq. (23) of Ref.[2] the activation energy is k*6733K = 0.580 eV and m=2.365. Using the relation (6) one obtains for T=273K the equivalent activation energy of 0.601 eV. It is easy to check that in the interval from -30° to +30°C the temperature dependences (5) for m = 2.365, E_a = 0.580 eV and for m=3/2, E_a = 0.601 eV differ by less than 1%.

The activation energy values in Eqs. (21, 22) of Ref.[2] are 0.605 and 0.603 eV respectively. Combining them with the value of 0.601 eV obtained above one gets the average experimental value of E_a = 0.603 ± 0.002 eV where the uncertainty covers all three experimental values. Therefore the experimental value for the effective gap energy is

$$E_{ef} = 2E_a = 1.206 \pm 0.004 \text{ eV}. \tag{7}$$

This result looks incompatible with the experimental values of E_g, which according to Table 1 of Ref.[2] are 1.1242 eV at 300K and 1.1367 eV at 250K. Note however that temperature independent E_{ef} should also incorporate temperature dependence of the band gap E_g.

Consider the exponential term of Eq.(2)

$$f(T) \propto exp(-E_g/2kT) \tag{8}$$

with E_g itself a function of temperature. The relative gradient of this function is

$$(df/f)/(dT/T) = E_g(T)/2kT - (dE_g/dT)/2k \tag{9}$$

If the temperature dependent E_g is replaced in Eq.(8) by a constant parameter E_{ef} then the relative gradient will be

$$(df/f)/(dT/T) = E_{ef}/2kT \tag{10}$$

Requiring the gradients given by Eqs. (9) and (10) to be equal at a given temperature, T, one gets for E_{ef}:

$$E_{ef} = E_g(T) - T\,(dE_g/dT) \tag{11}$$

Consider now the situation when in some temperature interval the band gap dependence can be approximated by a linear function

$$E_g(T) = E_0 - \alpha\, T\,. \tag{12}$$

Here E_0 is the band gap value extrapolated to T = 0K. In this case the effective energy gap found from Eq. (11) is

$$E_{ef} = E_0 - \alpha\, T - T\,(-\alpha) = E_0 \tag{13}$$

independent of T (and also of α). This conclusion can be verified by direct substitution of E_g in Eq.(8) by relation (12):

$$A\cdot exp(-E_g(T)/2kT) = A\cdot exp(-E_0/2kT + \alpha/2k) = A'\cdot exp(-E_0/2kT) \tag{14}$$

For temperatures above 250K and below 415K the parameterisation (12) is readily available in Ref. [2], Eq.(17). The value of E_0 in this equation is 1.206 eV in perfect agreement with the experimentally found E_{ef} presented in eq. (7). Thus there is no contradiction between the temperature dependent band gap energy and the temperature independent effective energy. To some extent surprisingly the effective gap value happens to be outside the range of the actual band gap values in the considered temperature interval.

For non-irradiated sensors it is usually assumed that the current is generated via traps with energy levels near the mid gap i.e. the $I(T)$ is described by Eq.(4) with Δ=0 and either temperature dependent $E_g(T)$ or the effective gap E_0. However in non-irradiated sensors the bulk generation current is typically quite small and presents no practical problems. The high current in such sensors is usually due to other reasons e.g. soft breakdown, which makes the above analysis irrelevant. It is more appropriate for irradiated sensors where the bulk current often dominates. The information on $I(T)$ for irradiated sensors is rather scarce. The survey [3] made in 1994 produced the effective gap value of 1.24 ± 0.06 eV, which agrees with the mentioned above E_0 = 1.21 eV for n_i, but has a substantial uncertainty. The study of ATLAS SCT sensors irradiated by ~3 10^{14} 24 GeV protons/cm^2 performed in 1997-99 resulted in the effective gap value of 1.21 eV [4]. Thus until proven otherwise the same temperature dependence with E_{ef} equal to 1.21 eV may be used for both irradiated and non-irradiated sensors.

In conclusion, the temperature dependence of the bulk generation current can be described as

$$\boldsymbol{I(T) \propto T^{2} \exp(-1.21eV/2kT)}$$

both for non-irradiated and irradiated sensors. The difference between the effective energy value and the actual gap energy is due to using temperature independent E_{ef} instead of the temperature dependent $E_g(T)$.

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References

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